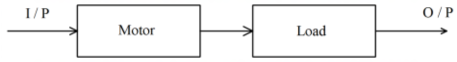
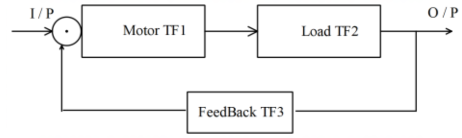
CONTROL

System

For any system it can be either open or closed

Open	Closed
	

Then we represent it with differential equations

We transfer these equations from **time domain** to the **S domain** using **<u>Laplace Transform</u>**

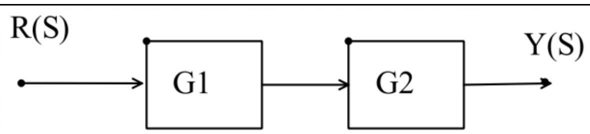
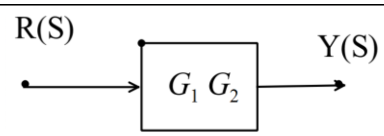
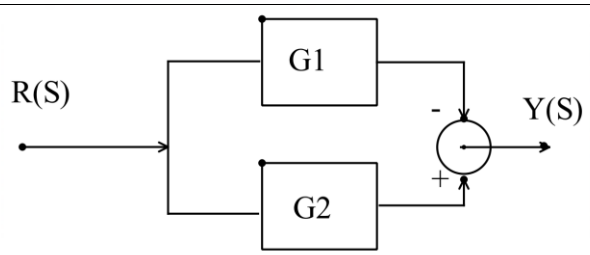
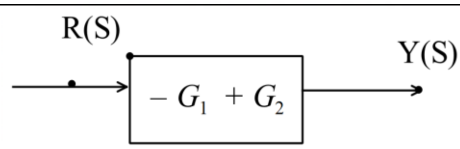
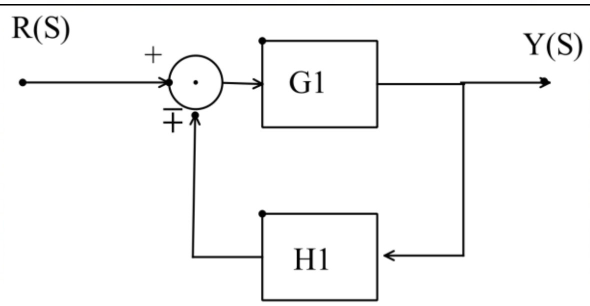
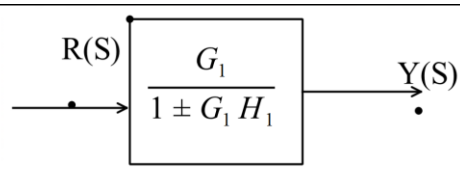
Then we get the Transfer Function $T.F = \frac{O/P(S)}{I/P(S)}$

Block Diagram

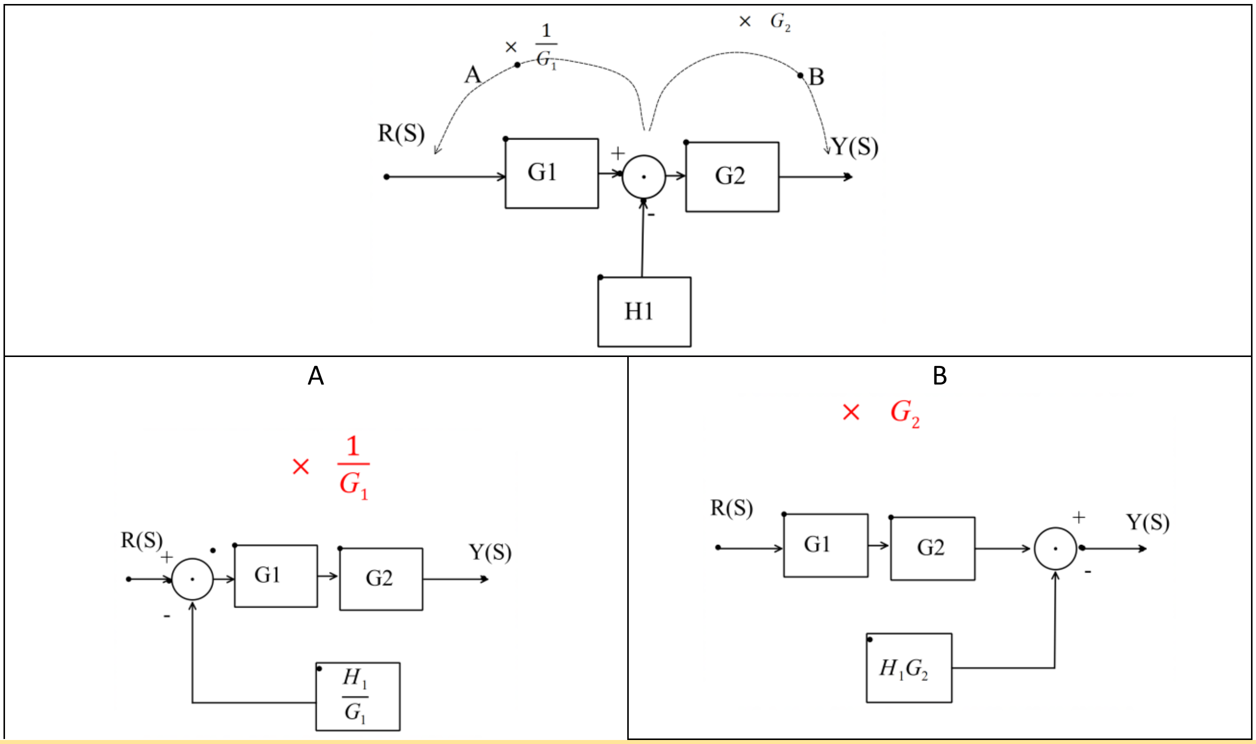
1- Components
2- Summing Point
3- Branching Point
4- Feed Back , Forward

Component Reduction

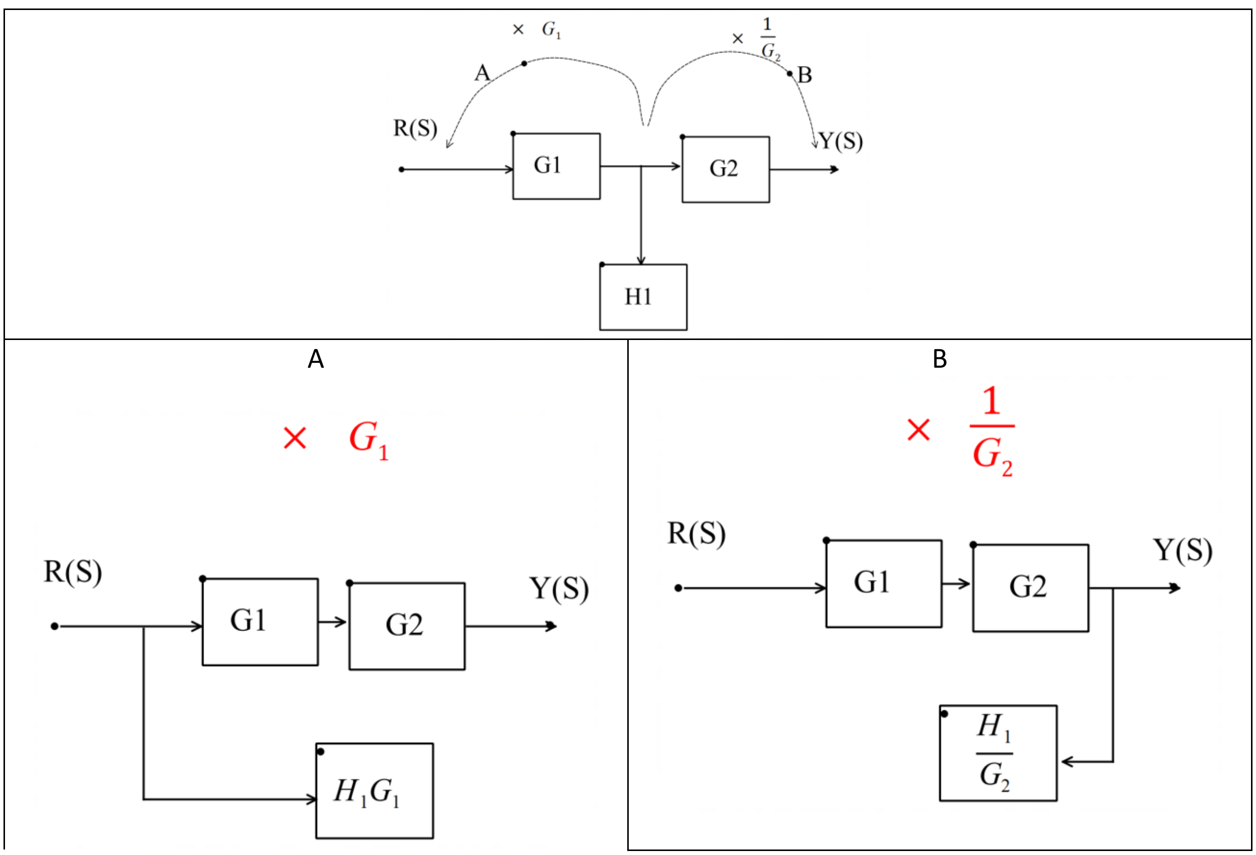
Has 3 Forms used to simplify block diagrams

Form	Reduction
	
	
	

Summing



Branching



Signal Flow graph

1- Every gain is put as $\xrightarrow{G_1}$
2- Every node is represented by a number
3- We count loops and type them according to numbers i.e L1 : 1231 ; L2 : 3453 etc.
4- $number\ of\ loops \geq number\ of\ feedbacks$
5- Get Δ as

$$\Delta = 1 - \sum L_i + \sum L_i L_j - \sum L_i L_j L_k + \cdots$$

as L_i , L_j $are\ loops\ that\ doesn't\ touch$
as L_i , $L_j . L_k$ $are\ loops\ that\ doesn't\ touch$

6- Get P and Δ_i

P is every main branch (from input to output)

$$assume\ there\ are\ 2\ Ps\ \ p_1 and\ P_2$$
$$\therefore \Delta_1 = \Delta_{P_1=0}$$
$$\Delta_2 = \Delta_{P_2=0}$$
$$meaning\ any\ loop\ in\ \Delta\ that\ touchs\ P\ will\ be = 0\ in\ \Delta$$

7- Get TF

$$T.F = \frac{\sum P_i \Delta_i}{\Delta}$$

Modelling

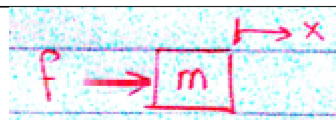
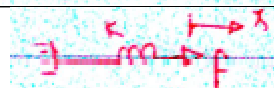
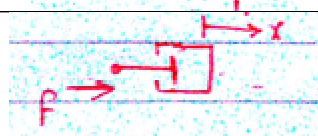
Electrical

Element	V	$V(S)\{Laplaced\}$
R	IR	R
L	$L\frac{di}{dt}$	LS
C	$\frac{1}{C}\int_0^t i\,dt$	$\frac{1}{CS}$

After you changed any electric circuit to S domain
just use **<u>Voltage Divider</u>** Rule and you will be fine

Mechanical (Linear)

Three types of forces

m		$F = m\,\ddot{x}$
K		$F = K(x_1 - x_2)$
B		$F = B(\dot{x}_1 - \dot{x}_2)$

- Study Every M separately then get an equation
- x_1 is the nearer to the M
- $x \to X$, $\dot{x} = SX$, $\ddot{x} = S^2X$
- Solve equations together

Rotational

Same as Linear (same equations and procedures) but difference in

m	J
x	θ
F	T

Complex System

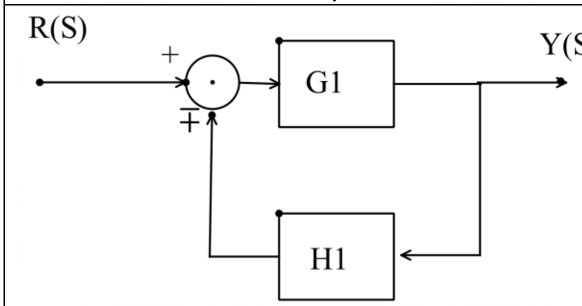
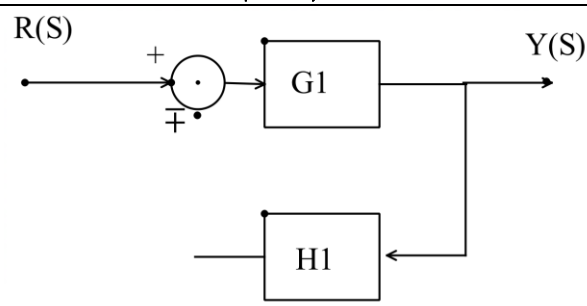
For a Complex system we study **Variables** and then **Relations (Equations)**
(we want $no.var = no.eqs + 1$)

Then by **<u>substituting</u>** we get the TF of System

Or : use **<u>block diagram</u>**

Means that the input is within limit (Bounded) so the output does not go to ∞

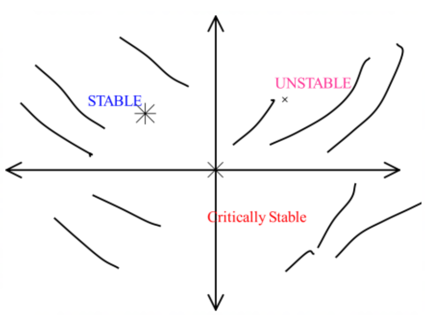
Ex: $e^{-t}\ (at = \infty) = \ 0 \rightarrow stable \quad but\ y = t\ (at\ t = \infty) = \infty \rightarrow unstable$

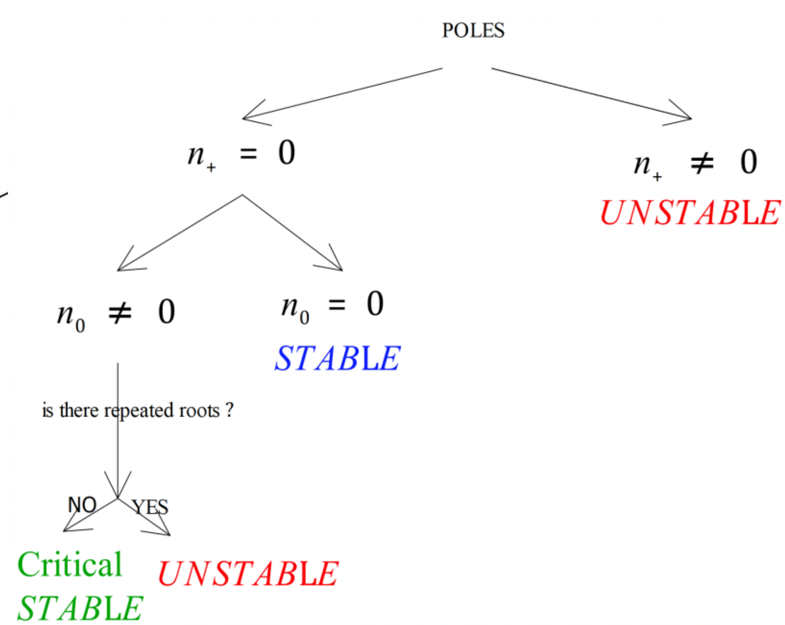
Closed System	Open System
	
$T.F = \frac{Y(S)}{R(S)} = \frac{G(S)}{1 + GH(S)}$ $G(S) = 0 \rightarrow Zeros$ $1 + GH(S) = 0 \ \rightarrow Poles\ (ch/c\ eqn.)$	$T.F = H(S)G(S)$ $closed\ loop \frac{ch}{c} = 1 + T.F_{open\ loop}$

Two ways to get stability

(1) Solve the TF equation in S domain (gets poles then n_+ , n_0 , n_-
(2) Use Routh method (gets n_+ , n_0 , n_- directly)

After you get poles test :





Routh Method

$$\boxed{S^4 + 2S^3 + S^2 + 4S + 2 = 0}$$

أبراسيت

	S^4	1	1	2
	S^3	2	4	0
$\overline{2}$	S^2	A_1 = −1	A_2 = 2	
$\overline{-1}$	S^1	B_1 = 8	0	
$\overline{4}$	S^0	2		

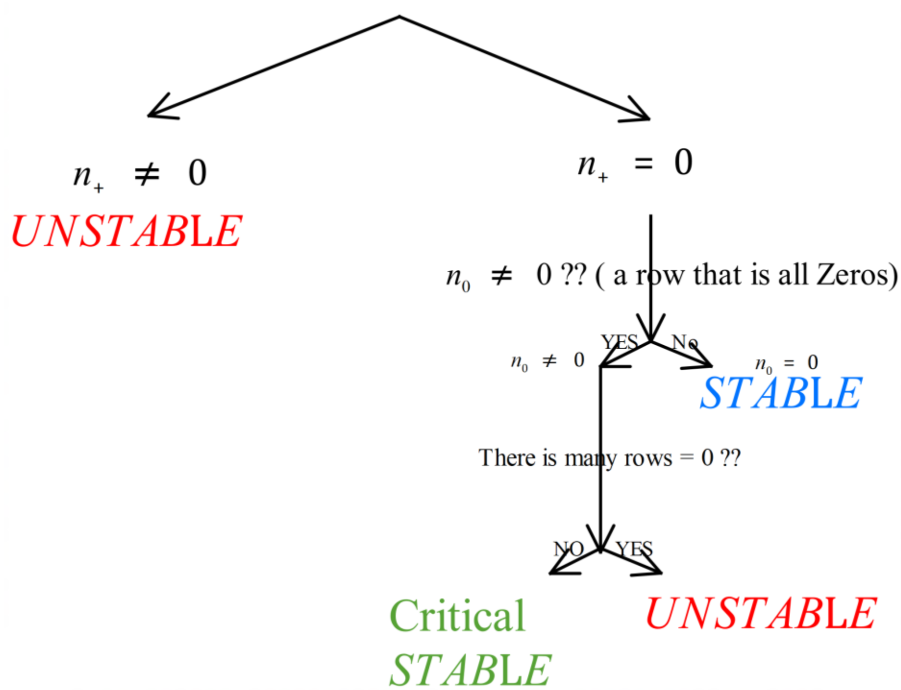
اكر الحلقة

معاملات الأس الزوجي +

معاملات الأس الفردي

Check
هو اكر الحلقة

n_+ = number of sign changes in first column



Distribution of

$n = the\ largest\ power\ in\ equation$

$$n = n_+ + n_0 + n_-$$
$$we\ get\ n_+$$
$$of\ there\ is\ no\ zero\ rows\ n_0 = 0$$
$$\therefore \boldsymbol{n_- = n - n_+}$$

$\boldsymbol{n_-}$	$\boldsymbol{n_0}$	$\boldsymbol{n_+}$
2	**0**	**2**

If there is a zero in a row (shifting)

We shift left to replace the zero with a number

$0 \quad -2$

$\xleftarrow{\text{Shift Left (1 time)}}$

$-2 \quad 0$

Notes :
لو الشفت فردي نضرب في سالب يعني

$-(-2 \quad 0)$

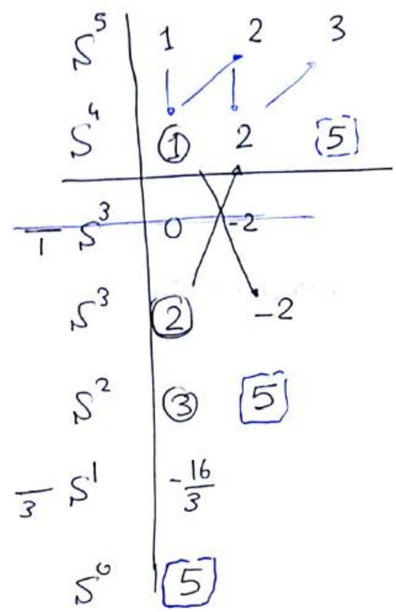
$2 \quad 0$

$2 \quad 0$

$+\,0 \quad -2$

Notes :
نطرح الصف القديم من الصف الجديد والناتج يوضع لنكمل به المسأله

$2 \quad -2$



Notes

- لو الشفت موجب نسيبه زي ما هو ونطرح برضه
- عدد مرات الشفت يعتمد لحد ما نحط رقم غير الصفر في اول عمود

n_+	n_0	n_-
2	0	3

A row of Zeros

$S^5 + 3S^4 + 2S^3 + 6S^2 + S + 3 = 0$

When we get a zero row

We look at the row before it

We call it $A(S) = 3S^4 + 6S^2 + 3$

Diff. $\frac{dA(S)}{dS} = 12S^3 + 12S$

NOTE : ممكن اقسم صف كامل علي رقم

We replace the zero row with 12 12 or 1 1

Again

$$B(S) = 3S^2 + 3$$
$$\frac{dB(S)}{ds} = 6S$$

NOTE :

دائما الصف اللي كله اصفار بيكون فردي و المعادله المساعده اللي فوقيه زوجيه

S^5	1	2	1
$A(S)$ S^4	3	6	3
S^3	~~0~~	~~0~~	
S^3	~~12/12~~	~~12/12~~	
S^3	1	1	
S^2	3	3	
S^1	~~0~~	~~0~~	صف آخر كله أصفار
S^1	6		
S^0	3		

How to get n_0

We get from the equation $P = P_+ + P_0 + P_-$

Where

$$P = the\ largest\ power\ of\ \boldsymbol{the\ first}\ aux\ equation$$

$$P_+ = P_- = no\ of\ change\ of\ sign\ \boldsymbol{after\ the\ aux\ equation}$$

$$\boldsymbol{so\ we\ can\ get:\ P_0 = n_0}$$

General Notes

1- If there is a missing term $S^3 + S + 2 = 0\ \ (S^2)\ \ 1 - don't\ forget\ to\ put\ it's\ coef. = 0\ 2 - UNSTABLE$
2- If the equations sign changes $S^3 + 2S^2 - S + 1 = 0\ \ then\ it's\ UNSTABLE$

Condition for stability

We solve with the k
we get the conditions
- no change in the sign of first column (all values +ve)
- all signs of main equation are the same (all terms +ve) (the last term = +ve)

$$S^4 + 25S^3 + 15S^2 + 20S + K = 0$$ Find K → stable

③ $K > 0$

(+)	S^4	1	15	K
(+)	S^3	25	20	
(+)	S^2	14.2	K	
① $0 <$	S^1	$\frac{284-25K}{14.2}$		
② $0 <$	S^0	K		

Stable
↓
$n_+ = 0$

عدد التغير في الإشارة في العمود الأول = 0

① $\frac{284-25K}{14.2} > 0 \Rightarrow K < 11.36$

② $K > 0$

③ $K > 0$

بتجمع الثلاث شروط أن

$\boxed{0 < K < 11.36}$ → stable.

Critical ← يعني تامر الحدود بتساعة ال K

يعني ($K = 11.36$) لو عوضت بيه هيدين أصفار ،

un stable ← $\boxed{0 > K > 11.36}$ → Range برا خارج

---

② $S^4 + S^3 + S^2 + 3S + K = 0$ K → stable.

(+)	S^4	1	1	K
(+)	S^3	1	3	
(−)	S^2	−2	K	
$0 <$	S^1	$\frac{-6-K}{-2}$		
$0 <$	S^0	K		

Un stable

stable
↓
$n_+ = 0$
↓
لا يوجد تغير اشارة في العمود الأول .

ولكن النظام un stable.

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ال (K) مش هتأثر لو حددت ال Poles ← سالبة (−2)

يعني النظام unstable

أياً كانت قيم (K) لأن عندي اشارة سالبة

State Space

In electrical circuit $\boldsymbol{states\ =\ No\ of\ Cs\ +\ No\ of\ Ls}$

In TF $\boldsymbol{states = degree\ of\ the\ denominator}$

We get state space matrices from

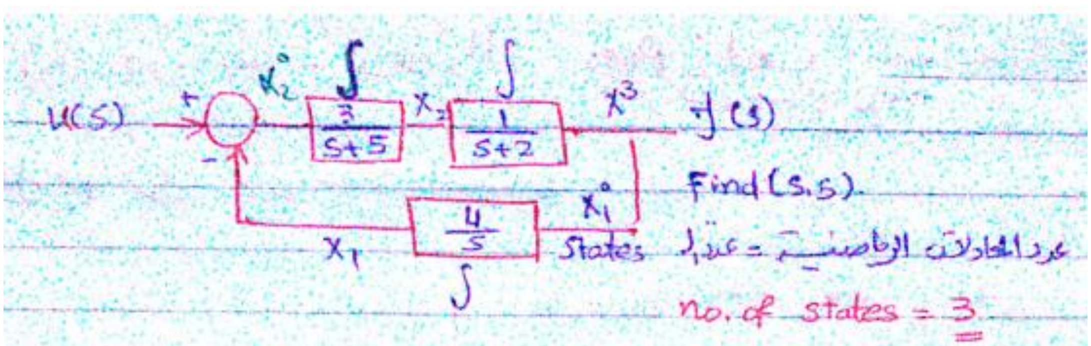
1. Differential Equation (time domain)
2. Block Diagram (S domain)
3. Signal Flow Diagram (S domain)
4. Transfer Function (S domain)

From Block Diagram

$$States\ (n) = Number\ of\ Equations\ (integrals\ \frac{1}{S})$$

$$\begin{bmatrix} \dot{x_1} \\ \dot{x_2} \\ . \\ . \\ . \\ \dot{x_n} \end{bmatrix} = \overbrace{\begin{bmatrix} & & \end{bmatrix}}^{\text{A\_n*n}} \begin{bmatrix} x_1 \\ x_2 \\ . \\ . \\ . \\ x_n \end{bmatrix} + \overbrace{\begin{bmatrix} . \\ . \\ . \end{bmatrix}}^{\text{B\_n*1}} u(t) \ \ ; \ y(t) = [\ ,\ \ ,\ ] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

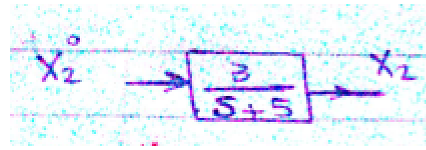
Example



1- We conclude the rest of variables from x_2

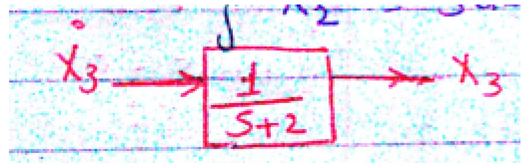
2- We study every gain individually

$$\frac{X_2}{\dot{X}_2} = \frac{3}{S+5} \rightarrow \quad from\ sum\ circle \rightarrow \ \dot{X}_2 = u - X_1$$



$\therefore$ طرفين في وسطين $\rightarrow SX_2 = 3\dot{X}_2 - 5X_2 = 3(u - X_1) - 5X_2$

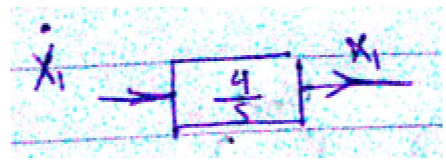
$laplace\ inverse \rightarrow \dot{x_2} = 3u - 3x_1 - 5x_2 \ \rightarrow \ (1)$



$$\frac{X_3}{\dot{X}_3} = \frac{1}{S+2}$$

$$SX_3 = \dot{X}_3 - 2x_3 = X_2 - 2X_3$$

$$L^{-1} \rightarrow \dot{x_3} = x_2 - 2x_3 \quad \rightarrow \ (2)$$



$$\frac{X_1}{\dot{X}_1} = \frac{4}{S} \ \rightarrow \ SX_1 = 4\dot{X}_1 = 4X_3$$

$$L^{-1} \rightarrow \ \dot{x_1} = 4x_3 \ \rightarrow \ (3)$$

$$y = x_3 \ \rightarrow \ (4)$$

So we get the State space

$$\begin{bmatrix} \dot{x_1} \\ \dot{x_2} \\ \dot{x_3} \end{bmatrix} = \overbrace{\begin{bmatrix} 0 & 0 & 4 \\ -3 & -5 & 0 \\ 0 & 1 & -2 \end{bmatrix}}^{A\_n*n} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \overbrace{\begin{bmatrix} 0 \\ 3 \\ 0 \end{bmatrix}}^{B\_n*1} \ u(t); \ y(t) = [0\ 0\ 1] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

From differential equation

The order of the highest diff. = no of states

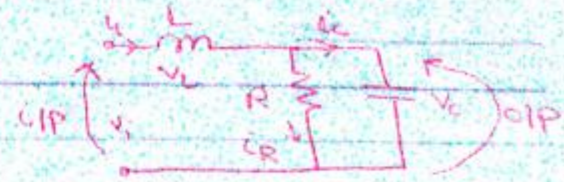
$$y''' + 6y'' + 11y' + 6y = 6\,u$$

$$y''' = \dot{x_3}$$
$$y'' = \dot{x_2} = x_3 \ \rightarrow \ (1)$$
$$y' = \dot{x_1} = x_2 \ \rightarrow \ (2)$$
$$y = x_1 \ \rightarrow \ (3)$$

$$\dot{x_3} = \ -6x_3 - 11x_2 - 6x + 6u(t) \ \rightarrow \ (4)$$

$$\begin{bmatrix} \dot{x_1} \\ \dot{x_2} \\ \dot{x_3} \end{bmatrix} = \overbrace{\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{bmatrix}}^{A\_n*n} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \overbrace{\begin{bmatrix} 0 \\ 0 \\ 6 \end{bmatrix}}^{B\_n*1} \ u(t) \ ; \ y(t) = [1\ 0\ 0] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$V_i = V_L + V_C$



$V_L = L\frac{di_L}{dt} = L\dot{X}_1$

$V_i = L\dot{X}_1 + X_2 \qquad \dot{X}_1 = \frac{V_i - V_2}{L} \rightarrow (1)$

no. of states = no. of (L) + no. of (C) = $\boxed{2}$

$x_1 = i_L$, $\dot{x}_1 = \frac{di_L}{dt}$

$x_2 = V_C$, $\dot{x}_2 = \frac{dV_C}{dt}$

$V_L = L\frac{di}{dt}$

$i_C = C\frac{dV_C}{dt}$

$i_L = i_R + i_C = \frac{V_C}{R} + C\frac{dV_C}{dt}$

$X_1 = \frac{X_2}{R} + C\dot{X}_2$

$\dot{X}_2 = \frac{X_1}{C} - \frac{X_2}{CR} \rightarrow (2)$

$$\begin{bmatrix} \dot{X}_1 \\ \dot{X}_2 \end{bmatrix} = \begin{bmatrix} 0 & \frac{-1}{L} \\ \frac{1}{C} & \frac{-1}{CR} \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} + \begin{bmatrix} \frac{1}{L} \\ 0 \end{bmatrix} V_i(t)$$

$$V_C(t) = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

$$L -> x = i_l\,;\dot{x} = \frac{di}{dt} \rightarrow v_l = L\,\dot{x} \quad but\ \ C \rightarrow \ \ x = v\ ,\dot{x} = \frac{dv}{dt} \rightarrow i_c = C\frac{dv}{dt} = c\dot{x}$$

From TF (transfer Function)

We’ve 4 forms

1- Controllable Form
2- Observable Form
3- Parallel Form (Diagonal)
4- Pole-Zero Form

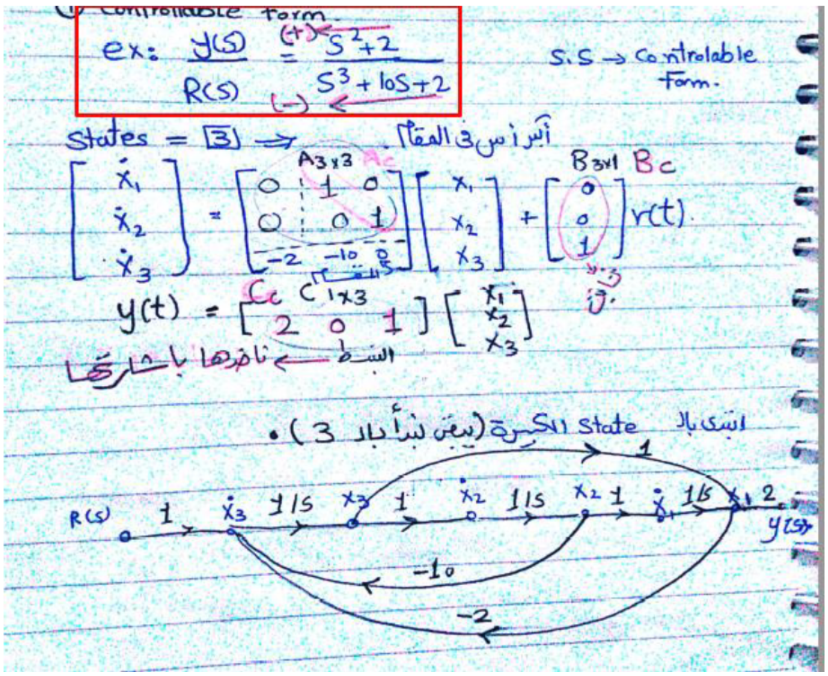
1- Controllable

$$5-\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \cdot \\ \cdot \\ \cdot \\ \dot{x}_n \end{bmatrix} = \overbrace{\begin{bmatrix} \square & \square & \square \\ \square & \square & \square \\ \square & \square & \square \end{bmatrix}}^{A\_n*n} \begin{bmatrix} x_1 \\ x_2 \\ \cdot \\ \cdot \\ \cdot \\ x_n \end{bmatrix} + \overbrace{\begin{bmatrix} 0 \\ 0 \\ 0 \\ \cdot \\ \cdot \\ 1 \end{bmatrix}}^{B\_n*1} u(t) \;\; ; \; y(t) = \overbrace{[\;,\;\;,\;]}^{C_{3*1}} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

A	B	C
اول عمود واخر صف افصلهم الصف الاخير : المقام من الاخر للاول واضرب في سالب شرط : معامل اكبر اس = 1 ومش هاخده	كلها اصفار ماعدا اخر عنصر =1	من معاملات البسط من الحد المطلق لاكبر حد (لو في تيرم ناقص بتحط معامه = 0)

في الرسم نبدأ باكبر state من الشمال

مثال :



2- Observable

$$3-\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \cdot \\ \cdot \\ \cdot \\ \dot{x}_n \end{bmatrix} = \overbrace{\begin{bmatrix} & & \\ & & \\ & & \end{bmatrix}}^{A\_n*n} \begin{bmatrix} x_1 \\ x_2 \\ \cdot \\ \cdot \\ \cdot \\ x_n \end{bmatrix} + \overbrace{\begin{bmatrix} \cdot \\ \cdot \\ \cdot \end{bmatrix}}^{B\_n*1} u(t) \;\; ; \; y(t) = \overbrace{[\;,\;\;,\;]}^{C_{3*1}} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

A	B	C
اخر عمود واول صف افصلهم الصف الاخير : المقام من الاخر للاول واضرب في سالب شرط : معامل اكبر اس = 1 ومش هاخده $A_O = A_C^T$	من معاملات البسط من الحد المطلق لاكبر حد (لو في تيرم ناقص بتحط معامه = 0) $B_O = C_C^T$	كلها اصفار ماعدا اخر عنصر =1 $C_O = B_C^T$

مثال

② observable.

$$\frac{Y(s)}{R(s)} = \text{نفس المسألة اللي فاتت}$$

A_o المقام B_o البسط

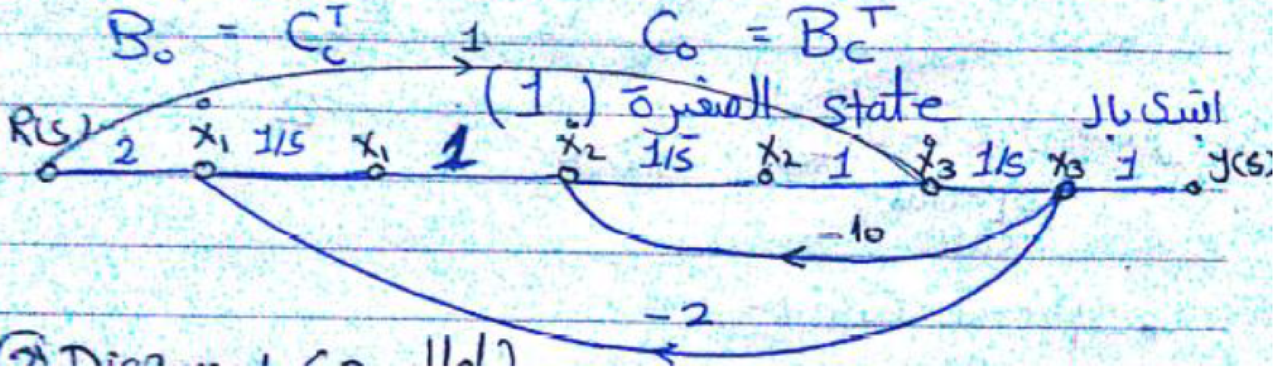
$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\end{bmatrix} = \begin{bmatrix}0 & 0 & -2\\ 1 & 0 & -10\\ 0 & 1 & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix} + \begin{bmatrix}2\\ 0\\ 1\end{bmatrix} r(t)$$

C_o

$$y(t) = \begin{bmatrix}0 & 0 & 1\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix}$$

$$A_o = A_c^T$$

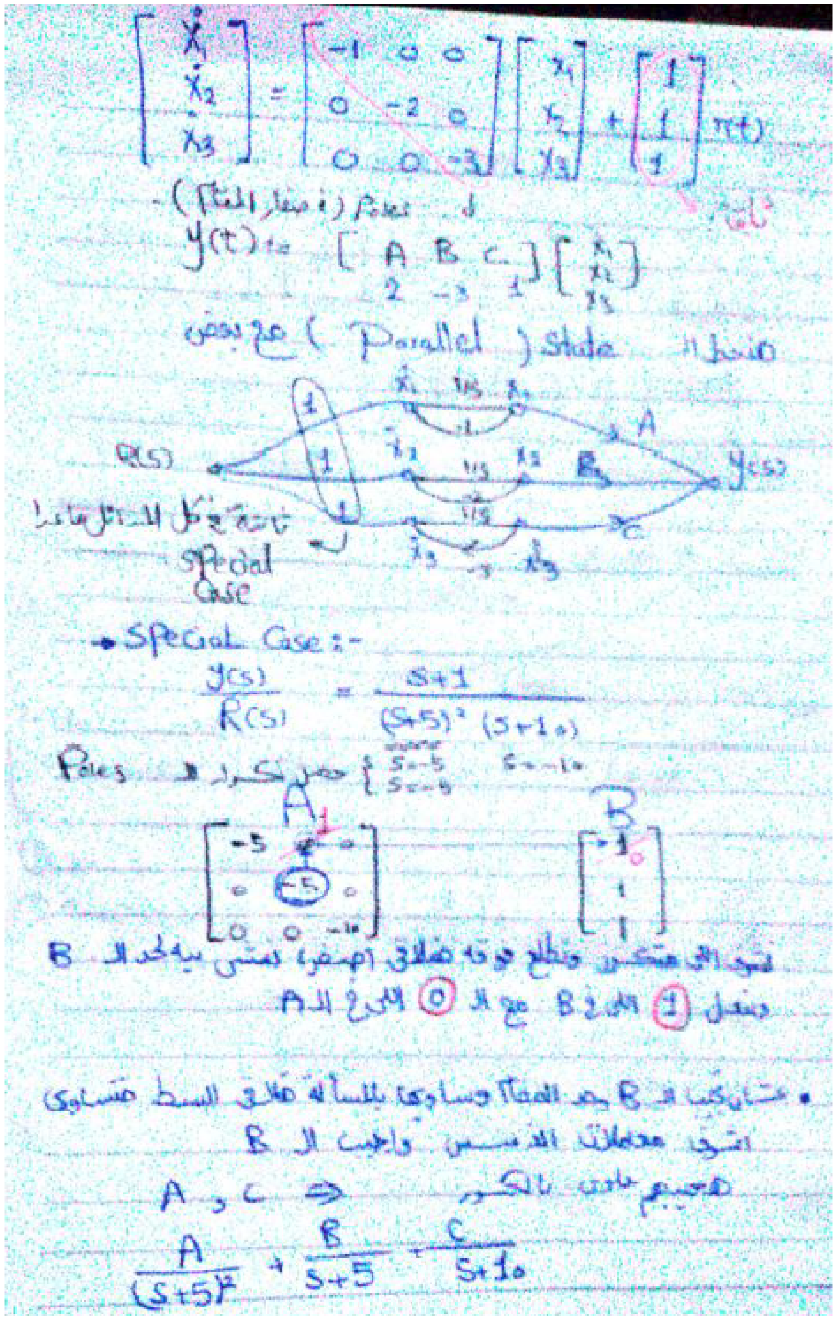
$$B_o = C_c^T \qquad C_o = B_c^T$$



Parallel (Diagonal) Form

A	B	C
نكتب الPoles من الصغير للكبير في القطر بتاع A والباقي اصفار	كلها 1111111111111111111111	ثوابت الكسور الجزئيه بترتيب ال A (صغير لكبير) الثواب كسور جزئيه - غير مكرر : مجرد النظر - مكرر : بيتوزع باسه مره ومن غير اسمه من غير اسه بتساوي معاملات

مثال



$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\end{bmatrix} = \begin{bmatrix}-1&0&0\\0&-2&0\\0&0&-3\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix} + \begin{bmatrix}1\\1\\1\end{bmatrix} r(t)$$

ثابتة

Poles ← (اصفار المقام) -

$$y(t) = \begin{bmatrix}A & B & C\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}$$

$A = 2,\ B = -3,\ C = 1$

فتحل الـ State (Parallel) مع بعض

ثابتة في كل الحالات ما عدا special case

• Special Case :-

$$\frac{Y(s)}{R(s)} = \frac{s+1}{(s+5)^2 (s+10)}$$

Poles ← حصل تكرار $\begin{cases} s=-5 \\ s=-5 \end{cases}$ $s=-10$

$$A = \begin{bmatrix}-5 & 1 & 0\\0 & -5 & 0\\0 & 0 & -10\end{bmatrix} \qquad B = \begin{bmatrix}0\\1\\1\end{bmatrix}$$

اللي هتتكرر هتطلع فوقه طالع (صفر) تمشي بيه لحد الـ B

هنحط (1) اللي في الـ B مع الـ (0) اللي في الـ A

• عشان نكتب الـ B بعد الفك وساويها بالمسألة نطلع البسط متساوي

اشتق معاملات الـ s واجيب الـ B

هجيبهم عادي بالكفر ⇐ A و C

$$\frac{A}{(s+5)^2} + \frac{B}{s+5} + \frac{C}{s+10}$$

Pole-Zero

---

(1)

بحلل المقام لاقواس

وبعدين اقسم درجات اولي ثم درجات تانيه

وبعدين احط البسطات : درجات اولي فوق الاولي لحد ما تخلص – درجات صفريه ع الباقي
والدرجات التانيه تتحط فوق التاني
(الشرط درجه البسط لا تزيد عن درجه المقام)

④ Pole Zero

$$\frac{Y(s)}{R(s)} = \frac{5(s+1)}{(s+2)(s^2+2s+3)}$$

لو المقام اتحلل حل كسور جزيئية لو مكالش :-

① $= \frac{s+1}{s+2}\left(\frac{5}{s^2+2s+3}\right)$ 1st 2nd

② S.F.G

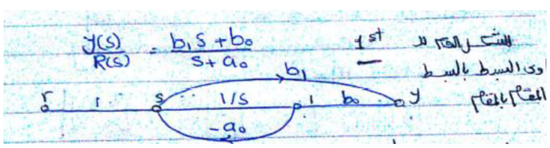
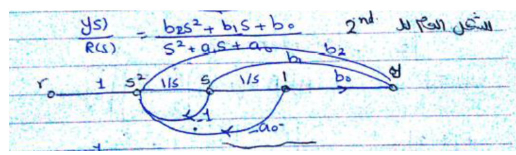
③ $\dot{x}_1 = \checkmark \quad \dot{x}_2 = \checkmark \quad \dot{x}_3 = \checkmark$

لازم اللى من الدرجة التانية يكون فوقه درجة تانية برضه نورم

④ حل بالماتركس

(2)

Sfg

First	Second
$\frac{Y(S)}{R(S)} = \frac{AS+B}{1S+C}$ Say $Y = AS + B$ $R = 1S + C$ Then draw sfg 	$\frac{Y(S)}{R(S)} = \frac{b_2S^2 + b_1S + b_0}{S^2 + a_1S + a_o}$ Say $Y = b_2S^2 + b_1S + b_0AS + B$ $R = S^2 + a_1S + a_o$ 

في المساله نحل الاتنين ونوصل بينهم series

$$S \xrightarrow{\frac{1}{S}} 1$$

$\dot{X_3} \xrightarrow{\frac{1}{S}} X_3$ نبدا الدرجه الاولي

والدرجه التانيه الباقي

(3)

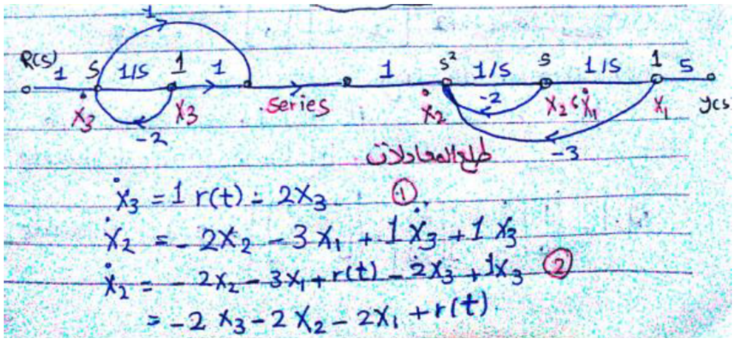
$\dot{x_1}$

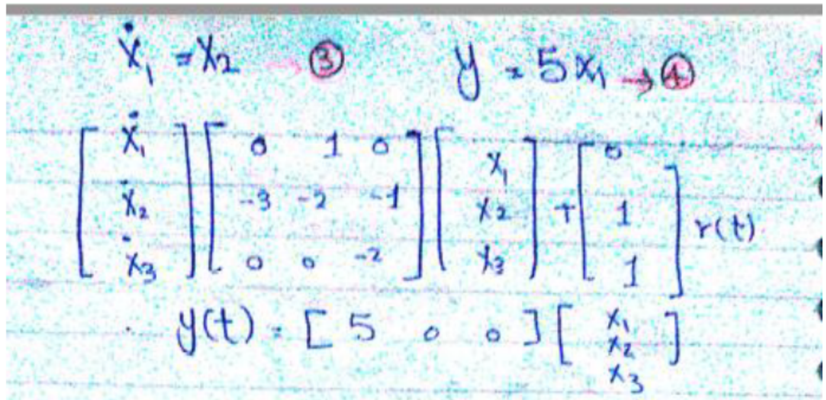
بعد ما نرسم

نجيب معادلات لكل من

$\dot{x_3}$, $\dot{x_2}$, $\dot{x_1}$ $y(t)$

ونختصر منهم ف بعض لحد ما تكمل المصفوفه بتاع state space





مساله مهمه

هديك قوانينها المهمه D:

$$1 - C\backslash C = |SI - A|$$
$$2 - T,F = C\ (SI - A)^{-1}B$$

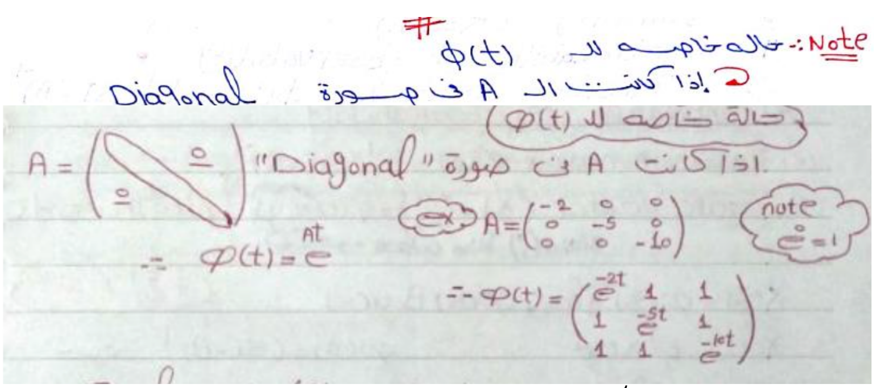
$$3 - M_C\ (controllable\ ) = [\ B\ \ AB]$$

$$M_O = \begin{bmatrix} C \\ CA \end{bmatrix}$$

IS	UN
$M_O \neq 0\ \ (OBSERVABLE)$ $M_C \neq 0\ (STABLE)$	$M_O = 0\ \ (UNOBSERVABLE)$ $M_C = 0\ (UNSTABLE)$

$$4 - Stability\ using\ Routh$$

$$5 - Transition\ Matrix\ \phi = L^{-1}[(SI - A)^{-1}]$$



$$6 - State\ Vector\ X(t) = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \phi(t)X(0) + \int_0^t \phi(t - \tau)Bu(\tau)d\tau$$

$$or\ better: X(S) = \phi(S)X(0) + \phi(S)BU(S)$$
$$then, x(t) = L^{-1}(X(S))$$

$$7 - Transformation\ Matrix\ (T)\ "\ to\ convert\ to\ diagonal"$$

<table>
<tr><th colspan="2">T</th></tr>
<tr><th>حاله خاصه
Controllable (A)</th><th>حاله عامه</th></tr>
<tr>
<td>
$T = \begin{pmatrix} 1 & 1 \\ P_1 & P_2 \end{pmatrix}$ 2 order

$T = \begin{pmatrix} 1 & 1 & 1 \\ P_1 & P_2 & P_3 \\ P_1^2 & P_2^2 & P_3^2 \end{pmatrix}$ 3 order

$A = \begin{pmatrix} 0 & 1 \\ -2 & -3 \end{pmatrix} \Rightarrow$ Controllable

$T = \begin{pmatrix} 1 & 1 \\ -1 & -2 \end{pmatrix}$

$\bar{A} = T^{-1}AT = \begin{pmatrix} 1 & 1 \\ -1 & -2 \end{pmatrix}^{-1} \begin{pmatrix} 0 & 1 \\ -2 & -3 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ -1 & -2 \end{pmatrix}$
$= \begin{pmatrix} -1 & 0 \\ 0 & -2 \end{pmatrix}$

$\bar{B} = T^{-1}B = \checkmark$
$\bar{C} = CT = \checkmark$

$T.F = \frac{2S+1}{S^2+3S+2}$
$(S+2)(S+1) = 0$
Poles $-2, -1$
</td>
<td>
الحالة العامه
$\begin{pmatrix} \dot{X}_1 \\ \dot{X}_2 \end{pmatrix} = \begin{pmatrix} -5 & -1 \\ 3 & -1 \end{pmatrix} X + \begin{pmatrix} 2 \\ 5 \end{pmatrix} u$
$, y = \begin{pmatrix} 1 & 2 \end{pmatrix} \begin{pmatrix} X_1 \\ X_2 \end{pmatrix}$

نفرض أن ال Poles
$P_1 = -2$
$P_2 = -4$
$T = \begin{pmatrix} V_{11} & V_{21} \\ V_{12} & V_{22} \end{pmatrix}$

at $P_1 = -2$
$(P_1 I - A)\begin{pmatrix} V_{11} \\ V_{12} \end{pmatrix} = 0$
$\left[\begin{pmatrix} -2 & 0 \\ 0 & -2 \end{pmatrix} - \begin{pmatrix} -5 & -1 \\ 3 & -1 \end{pmatrix}\right]\begin{pmatrix} V_{11} \\ V_{12} \end{pmatrix} = 0$
$3V_{11} + V_{12} = 0 \to 1$ بفرض
$-3V_{11} - V_{12} = 0 \to 2$
Put $V_{11} = 1$
$V_{12} = -3$

at $P_2 = -4$
$(P_2 I - A)\begin{pmatrix} V_{21} \\ V_{22} \end{pmatrix} = 0$
$\left[\begin{pmatrix} -4 & 0 \\ 0 & -4 \end{pmatrix} - \begin{pmatrix} -5 & -1 \\ 3 & -1 \end{pmatrix}\right]\begin{bmatrix} V_{21} \\ V_{22} \end{bmatrix} = 0$
$V_{21} + V_{22} = 0 \to 3$
$-3V_{21} - 3V_{22} = 0 \to 4$
Put $V_{21} = 1$
$V_{22} = -1$

$T = \begin{pmatrix} 1 & 1 \\ -3 & -1 \end{pmatrix}$
#
</td>
</tr>
</table>

Laplace

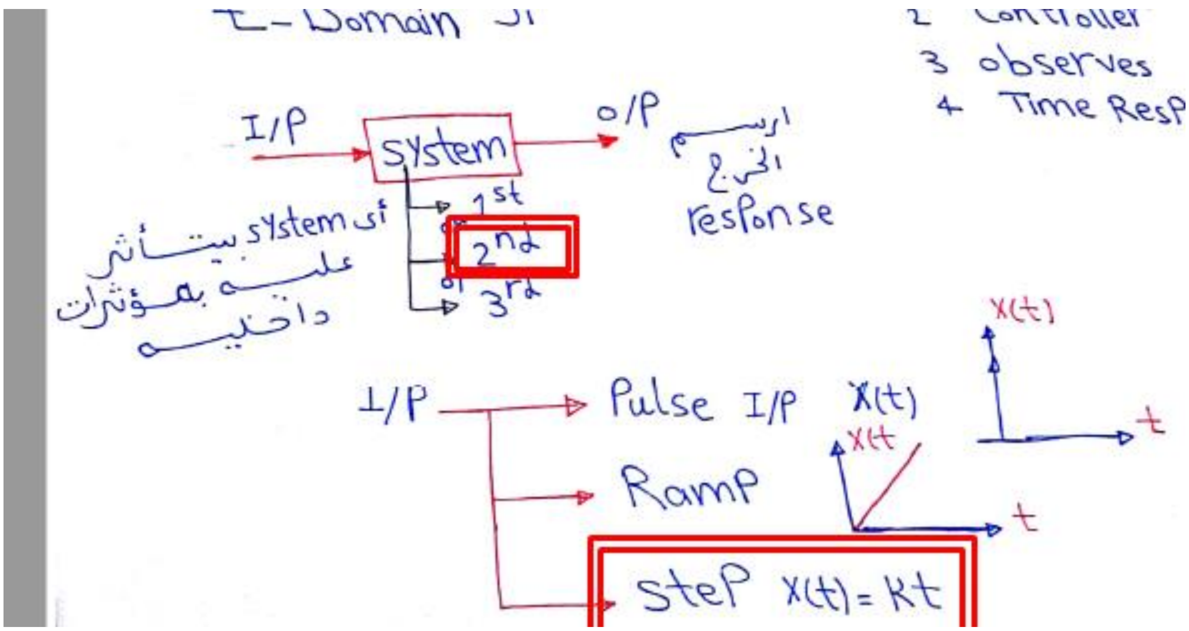
note	t	S
	1	$1/S$
	10	$10/S$
	$10e^{-2t}$	$\frac{10}{S+2}$

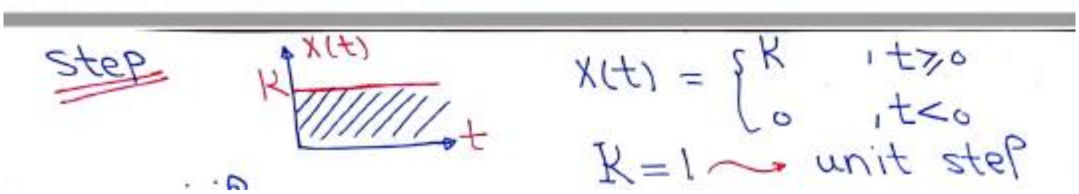
Time Response

System Properties we study are : Stability,Controlability,observability,Time Response

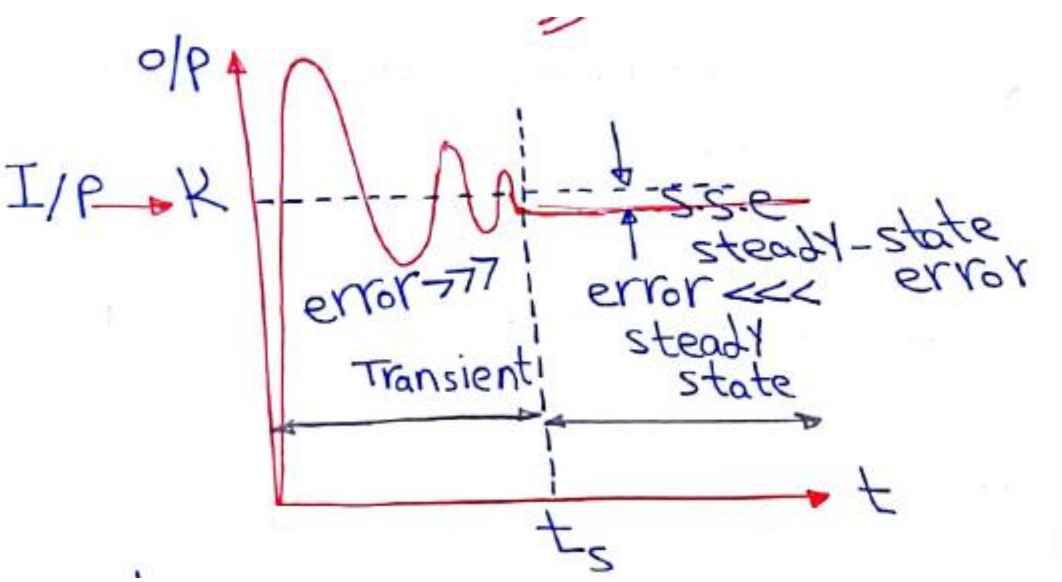
Time Response : Systems output in time domain

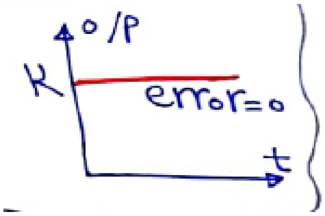
اشكال الممكنه للدرجه الدخل وشكله ولكننا هنشتغل على Step و second order





في هذه الحاله يكون شكل الخرج كاللآتي :





ملحوظه :

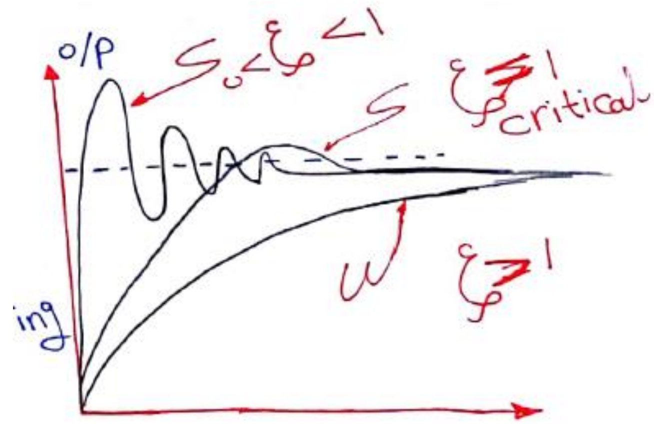
لو السيستم مثالي لن يحدث ترانزينت ولن يكون هناك ايرور

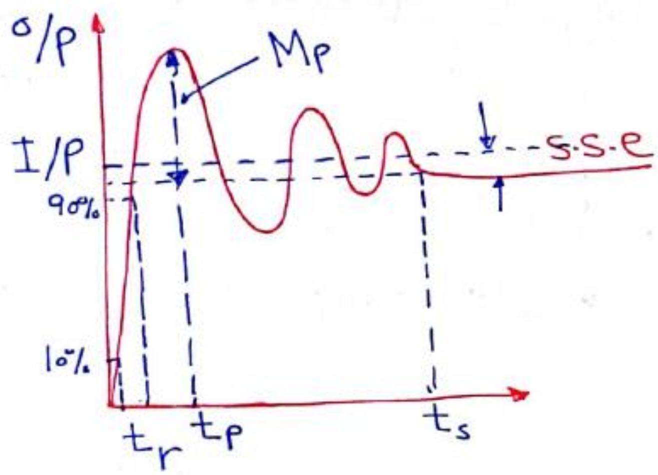
General Form

$$\boldsymbol{TF = \frac{\omega_n^2}{S^2+2\zeta\omega_n S+\omega_n^2}}$$

$$where : \omega_n : natural\ frequency \quad and \ \ \zeta : damping\ ratio$$

$Underdamping\ (quickest)(osc.)$	$0<\zeta<1$
$critical\ damping\ (\ lil\ Oscillation)$	$\zeta=1$
$Over\ Damping\ (\ slooow, no\ osc,)$	$\zeta>1$





Mp أعلى ذبذبه حصلت عنده

tp الوقت اللى عنده حصلت الذبذبه

tr الوقت اللازم للـ system ان يتحرك من 10% الى 90%

$$M_p\ (maximum\ overshoot) = e^{-\frac{\pi\zeta}{\sqrt{1-\zeta^2}}}$$

$$t_p = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}} = \frac{\pi}{\omega_d}$$

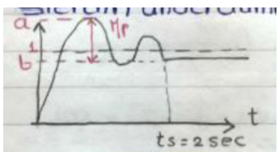
$$\omega_d = \omega_m\sqrt{1-\zeta^2} \rightarrow damping\ freq.$$

$$\phi^0 = \cos^{-1}(\zeta)$$

$$t_r = \frac{\pi-\phi}{\omega_d} \rightarrow \phi^r = \frac{\phi^0 * 3.14}{180}$$

$$t_s\ (\,error\ 2\,\%) = \frac{4}{\zeta\omega} \qquad or \quad t_s\ (\,error\ 5\,\%) = \frac{3}{\zeta\omega}$$

خطوات الحل



1- نتاكد انها على الصوره بتاع الاسطمبه اللي عندنا يعني لو في معامل لل S نقسم عليه

2- لو الحد المطلق مش زي البسط نضرب ونقسم البسط لحد ما يبقي زيه المهم نشتغل زي الاسطمبه

$$C(t) = 1 - \frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}} \sin(\omega_d t + \phi)$$

3- $M_p\,\% = \frac{a\text{-}b}{b}$

4- $C(\infty)\ final\ value\ (b)$

$S\ domain$	$time\ domain$
$c(\infty) = \lim_{S\to 0} S\ C(S)$ $C(S) = R(S)\,TF = \frac{\text{برا رقم زياده}}{S} * S * TF = \text{رقم زياده} * TF$	$put\ t = \infty$ $c(\infty) =$

5- $SSE = R(\infty) - C(\infty)$

Error Constants

Feedback = 1	Feed back = H(S)
$\boldsymbol{s.s.e = x(\infty) - y(\infty)}$	$s.s.e = \lim_{S\to 0} SE$ $E\ (error) = R(S) - H(S)\ \overbrace{Y(S)}^{=EG(S)}$ $\therefore E = \frac{R(S)}{1+G(S)H(S)}$ $\boldsymbol{s.s.e = \lim_{S\to 0} S\,\frac{R(S)}{1+G(S)H(S)}}$

STEP (1)

$$r(t) = k_p$$

$$s.s.e = \frac{K}{1+\lim_{s\to 0} G(S)H(S)} = \frac{1}{1+k_p}$$

$$\overbrace{k_p}^{\text{error constant}} = \lim_{s\to 0} G(S)H(S)$$

Ramp (2)

$$r(t) = k_v\,t$$

$$s.s.e = \frac{K}{\lim_{s\to 0} S\,G(S)H(S)} = \frac{1}{k_v}$$

$$\overbrace{k_v}^{\text{error constant}} = \lim_{s\to 0} S\,G(S)H(S)$$

Acceleration (3)

$$r(t) = k_a \frac{t^2}{2}$$

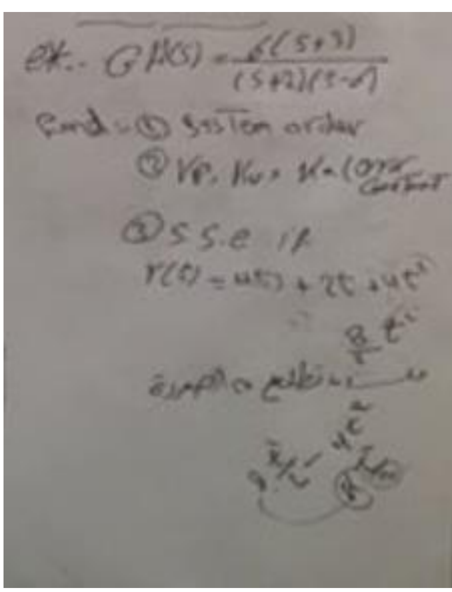
$$s.s.e = \frac{K}{\lim_{s \to 0} S^2\, G(S)H(S)} = \frac{1}{k_a}$$

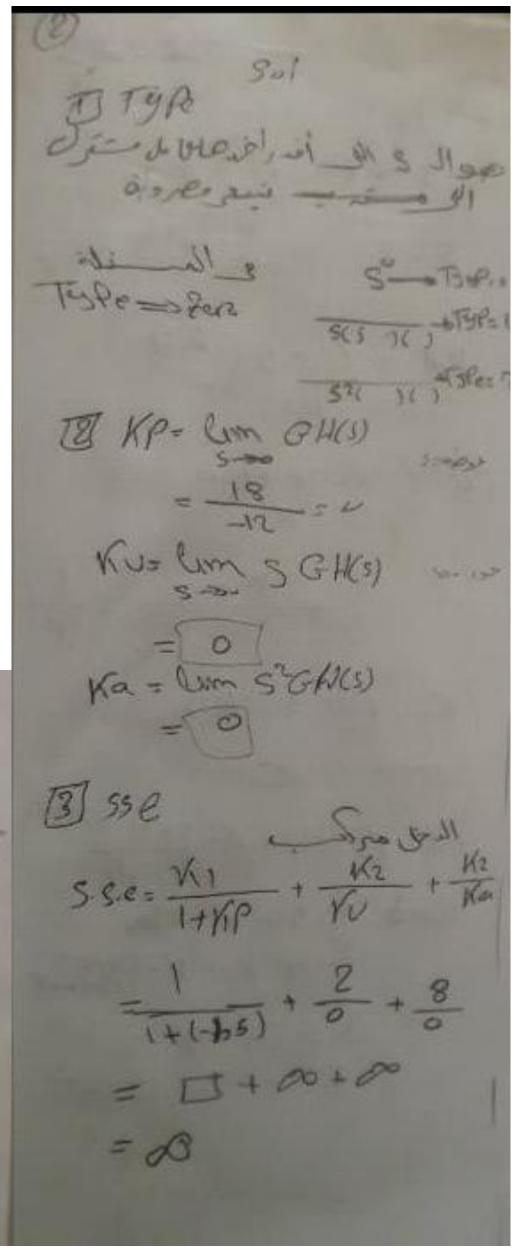
$$\overbrace{k_a}^{\text{error constant}} = \lim_{s \to 0} S^2\, G(S)H(S)$$

Type

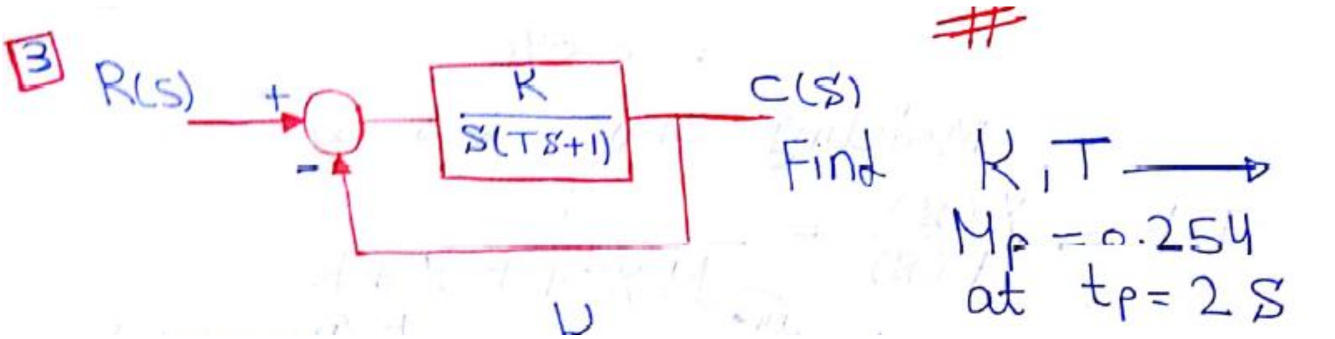
هو درجه (اس) ال S اللي اقدر اخدها مشترك من المقام

مثال

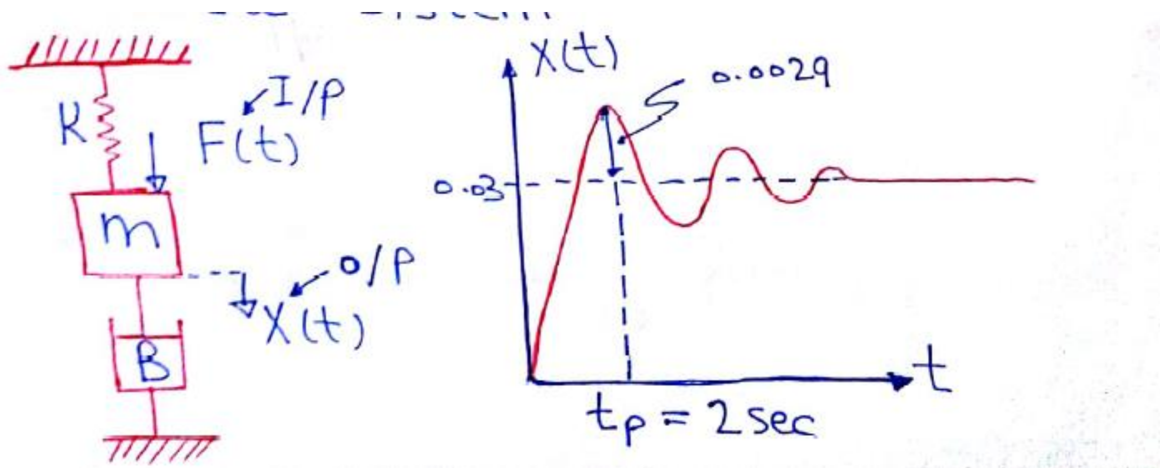




ممكن يجيب رسمه بلوك دياجرام تجيب منها ترانسفير فانكشن



ممكن يجيب ميكانيكا تجيب منها ترانسفير فانكشن



Root Locus

المحل الهندسي لمسار بين نقطه ال pole الي نقطه ال zero

$$T,F\ (Closed\ Loop) = \frac{G(S)}{1+G(S)H(S)}$$

$$Open\ Loop = G(S)H(S)$$

$$\therefore chc\ of\ closed = 1 + TF(open\ loop)$$

خطوات حل المساله

(1) تحديد معادله ال Open loop

نحدد معادله ال G(s)H(S) **بشرط** يكون ال Kمضروبه في البسط كله

If not {

- جيب معادله ال chc بتاع closed = 1+G(S)H(S)
- اقسم على اي حاجه مفيهاش k
- هيطلعلك 1+ 7aaga bastha madrob f K
- اذن G(S)H(S) = el 7aga de

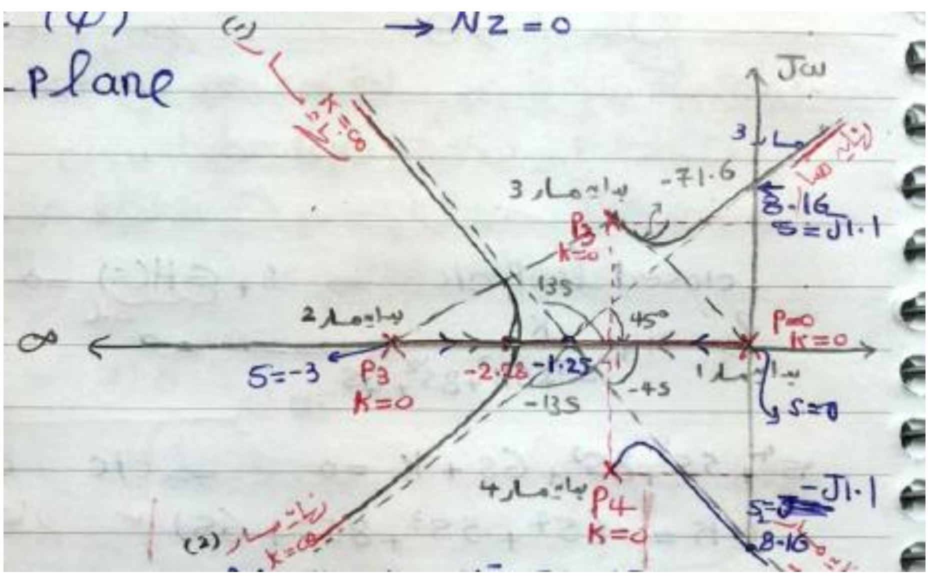
}

ثم نحل المعادله دي

اصفار البسط Zeros نرسمها في ال S planeبرمز $\boldsymbol{O}$ وعددها n_z

اصفار المقام Poles نرسمها في S plane برمز $\times$ وعددها n_p

(2) ارسم ال polesو ال zeros على S plane



(3) حدد القطع على ال Real Axis اللي هتاخدها (اول قطعه معانا – تاني مش معانا – وهكذا)

وارسم الاسهم خارجه من الPoles وداخله في ال Zeros او ال∞

(4) الخطوط التقاربيه لتحديد مسار ال poles التي ليس لها zero

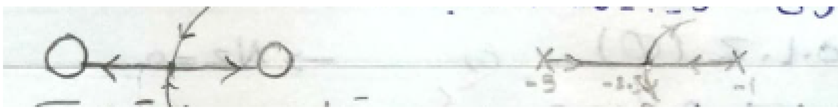
$$n_0(number\ of\ asympt.line\ ) = n_p - n_z$$

$$\delta(\text{الارتكاز}) = \frac{\sum P - \sum Z}{n_p - n_z}$$

$$\theta = \frac{(2l-1)180}{n_p - n_z} \quad (\ don't\ use\ it\ )$$

$$1{:}\,180\quad or\quad 2{:}\,90\,,-90\ or\quad 3{:}\,60,-60,180\ or\ 4{:}\,45,-45,135,-135$$

(5) حل مشكله وجود زوجين من نفس النوع



لايجاد نقطه التفريع

1-Get chc of closed loop = 1+G(S)H(S)

2-Simplify it to linear equation

4- Get K on one side

5-$\frac{dK}{dS} = 0$

6-Solve equation of step 5 to get point of تفريع

لازم تكون realو بين النقط

(6) تحديد زوايا مغادره وهبوط الcomplex poles

From Pole	To Zero
Departure	Arrive
$\theta_d = 180 - \sum\theta_p + \sum\theta_z$ $note : \theta = \tan^{-1}\frac{\text{مقابل}}{\text{مجاور}}$	$\theta_a = \sum\theta_p - \sum\theta_z - 180$

(7) ال Stability تقاطع ال root locus بالمحور الرأسي

1-From closed loop chc : we do routh

2-We get equation in $must\ f(k) > 0\ \ to\ \ be\ stable$ for the sign not to change get K = س مثلا

3-From $A(S)$ المعادله المساعده قبل الداله اللي ليها صفر we solve it and put (k= س)

فكره

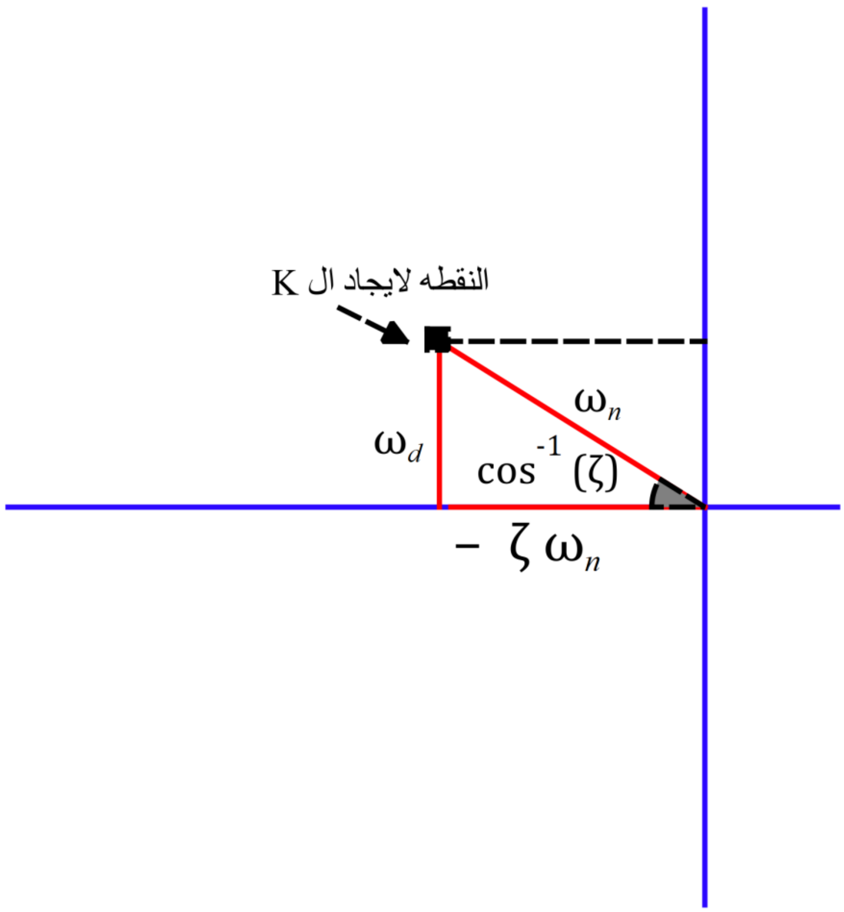
Find K

Giving

ω_n	ζ	t_s	ω_d
		$t_S = \frac{4}{\zeta\omega_n}$	

برسم المثلث نوجد النقطه

$$T,F = \frac{\omega_n^2}{S^2 + 2\zeta\omega_n S + \omega_n^2} \to S^2 + 2\zeta\omega_n S + \omega_n^2 = 0 \to S_{1,2} = -\zeta\omega_n \pm j\,\omega_d$$

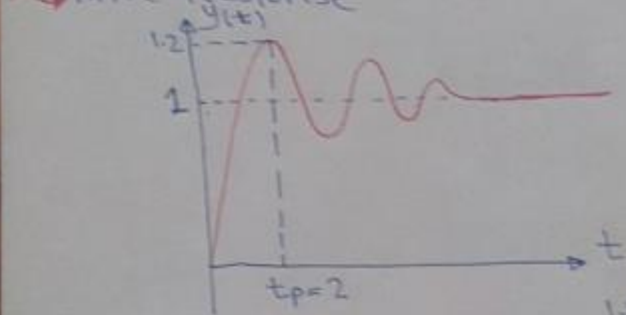


بعدها نقيس المسافات بين تلك النقطه و الpoles وا لzeros

$$k = \frac{l_{p_1} l_{p_2} l_{p_3}}{l_{z_1} l_{z_2} l_{z_3}}$$

افكار في امتحانات

~ Time Response



Find $T.F = \dfrac{\omega_n^2}{S^2 + 2\zeta\omega_n S + \omega_n^2}$

$0.2 = M_p = e^{-\pi\zeta/\sqrt{1-\zeta^2}}$

$\Rightarrow \zeta = ✓$

$= \dfrac{1.2 - 1}{1} = 0.2$

$t_p = \pi/\omega_d = \dfrac{\pi}{\omega_n\sqrt{1-\zeta^2}} = 2$

$\omega_n = ✓$

~ S.S

$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -5 & -10 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

$C = [\,0 \quad 0 \quad 2\,]$

Find ① $C/C = |SI - A|$

② $T.F = B(SI - A)^{-1}$ X

Controllable

$T.F = \dfrac{2S^2 + 5S + 0}{S^3 + 10S^2 + 5S + 1}$

لما يجيب اكتر من 2 في 2 اعرف ان فيها خدعه وهي انها كونترولابل

C/C $S^3 + 10S^2 + 5S + 1$

$T.F = \dfrac{2S}{S^2 + 4S + A}$ Find $A \longrightarrow \zeta_{critical}$

$4 = 2\zeta\omega_n$

$\zeta = 1 \longrightarrow \omega_n = 2 \Rightarrow A = 4$